Air and Space this Week

Item of the Week

Venus, Dichotomy, and the Size of the Solar System

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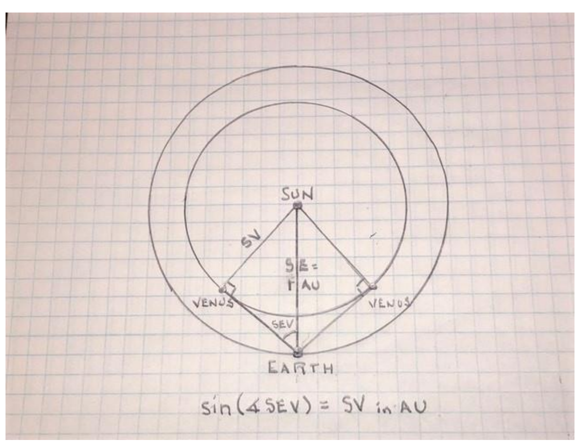
Simple trigonometry can be used to determine the size of the Solar System, especially when we see exactly half of Venus' illuminated hemisphere. Just ask some pre-telescope astronomers, Edmund Halley, or Captain Cook, but not Guillaume-Joseph-Hyacinthe-Jean-Baptiste Le Gentil de la Galaziere (aka Le Gentil)!

VENUS

The position of Venus in the sky, and the degree of illumination of its surface, can give astronomers a clue as to the size of the Solar System. I hope you've been looking at Venus in the morning sky of late. You can't miss it, as it is shining VERY brightly at magnitude -4.8; only the Sun and the Moon are celestial objects brighter than that. A few weeks ago, Venus was a thin crescent, but it has been shrinking in apparent size, but growing in phase. Keep watching (a small telescope will do) this week and you will see half its illuminated surface because we are looking at it directly from the side, like we see with a "Half Moon." This coincides, at least approximately, with Venus moving to the greatest distance in the sky from the Sun as it can get, which is coming up on **March 21**. The geometry of that situation is called "Dichotomy."

DICHOTOMY

Imagine that you can view the inner Solar System from "above." Earth follows a near-circular path, as does Venus, as shown in the diagram below.



Note the following. Look at the Earth-to-Venus line on either side of the diagram (the drawing is not to scale). When the line is tangent to the orbit of Venus, on either side, the Venus-Earth-Sun angle is at its greatest. Venus will appear closer to the Sun in the sky as seen from Earth if it is at any other point on its orbit (except the tangent point on the other side).

Now some simple geometry and trigonometry. Any line tangent to a circle is perpendicular to (at a right angle to) a radius of the circle at that point the line hits. On the diagram, note that the maximum Sun-Venus-Earth angle is a right angle (90°), making the Sun-Venus-Earth triangle a right triangle.

When Venus is at Dichotomy, if we measure the Venus-Earth-Sun angle, we can tell Venus' distance from the Sun, sorta. The sine of the VES angle is the ratio between the Sun-Venus distance to the Sun-Earth distance, which is called an "Astronomical Unit" or AU. Therefore, we know the Sun-Venus to be: sin (VES) x (1 AU)! We can do the same thing for Mercury. Thus, by observing and simple math, we know the size of the innermost Solar System***, at least in terms of the Sun-Earth distance***. These sorts of geometric observations were conducted as soon as telescopes good enough to determine phases accurately, by the mid/late 1600s.

OTHER METHODS TO DETERMINE THE SCALE OF THE SOLAR SYSTEM

Kepler's Third Law

Trigonometric observations weren't the only way for astronomers to determine the size of the Solar System in the 1600s. The telescope was invented in 1608, and was good enough that by the mid-century to allow dichotomy-based size estimations. At that same time, Johannes Kepler was working out his soon-to-be-famous Kepler's Three Laws of Planetary Motion. The Third Law is of interest here is, as he stated it, "… the ratio which exists between the periodic times of any two planets is precisely the ratio of the 3/2th power of the mean distances." And you thought ***I*** was wordy!

Simply put, the period of a planet's orbit squared is proportional to the radius of its mean orbit cubed. With a little math, this technique gave the distance of each known planet to the Sun, but ***still in terms of the AU***.

Astronomers lamented, "If only we knew the Sun-Earth distance itself, then we'd know the size of the Solar System and the distance each planet is from the Sun."

Venus Again

Well, there was a way to get actual distances in units other than the AU, at least on paper, and it used trigonometry, too. On rare instances, Mercury and Venus appear to travel across the face of the Sun, a phenomenon known as a "transit." You might remember the last Transit of Venus, in June, 2012, the last any of us will see in our lifetime. Venus was big enough to see with your protected but without magnification eye. Mercury transits occur a bit more often, but Mercury is so small you'd need a telescope to see it when it is in transit. If two or more astronomers could observe a transit, and record accurately the time when it started and when it ended, and the track it took across the Sun, it would be possible to work out the Sun-Earth distance in kilometers, miles, whatever.

Edmund Halley, of comet fame, figured out the mathematics involved. European astronomers tried to use Halley's technique for the 1761 Transit of Venus, but for a variety of reasons, this particular transit was unsuitable. Then Francis Wallis "discovered" Tahiti in 1767, just in time to pique the interest of the Royal Society and the Admiralty.

Observing planetary transits was one of the principal objectives of the first great voyage of exploration by Captain James Cooke. He was at Tahiti in 1679 and his astronomers determined the transit times as accurately as they could from a place that is called "Point Venus" to this day. [Point Venus is also "famous" as the debarkation point for the *HMS Bounty*, laden deeply with breadfruit and an unhappy crew.] There was a transit of Venus later that year, and Cook's team was in New Zealand and made measurements of it from a place still called "Mercury Bay."

A number of astronomers faced trials and tribulations to observe transits in order to determine Solar System scale. Alas, the precision required for success exceeded the capabilities of the timepieces and telescopes of that period, and the results were unimpressive. But the techniques were valid, the failure came from the technology level of the tools available at the time. That would soon be continuously refined as new technology was developed, a classic example of the synergy between the advancement of scientific understanding and the technology and tools that support the necessary observations. [Recall Copernicus knew an observational proof of the correctness of the heliocentric Solar System, again involving fair Venus. It involved the phases of illumination we see Venus go through. In the geocentric model of the Solar System, Venus would be backlit, or nearly so, as seen from Earth. That means that if we saw Venus, it would show only crescent phases. If Venus showed a full range of lunar-like phases, then Copernicus' heliocentric model would be correct. Alas, lacking a telescope capable of showing the range of phases on Venus, he couldn't use his test; see here. But astronomers having a telescope answered some questions, but others arose, as did the technology necessary to answer them. As Science marches on!]

REFERENCES

Venus at Dichotomy

https://profmattstrassler.com/articles-and-posts/relativity-space-astronomy-and-cosmology/transit-of-venus-and-the-distance-to-the-sun (includes Venus parallax)

"Debate Over Dichotomy," March 2022 issue's Observing section on Celestial Calendar, *Sky and Telescope* magazine, AAS Sky Publishing, p. 50.

Lunar and Planetary Institute; *The Geology of Terrestrial Planets*, chapter 4: https://www.lpi.usra.edu/publications/books/geologyTerraPlanets/4_Venus.pdf (p. 58)

Phys.org piece on "How Did Ancient Civilizations Make Sense of the Cosmos, and What Did They Get Right?" here: https://phys.org/news/2022-03-ancient-civilizations-cosmos.html; see the second paragraph in the "Getting it right – sometimes" section. The third and fourth

paragraphs of that section is about Eratosthenes and his determination of the circumference of the Earth, a topic I covered in an earlier Item of the Week.

Kepler's Third Law

Who Measured the Solar System – and How?
https://eclipse.gsfc.nasa.gov/transit/HalleyParallax.html

For Science Teachers: https://pwg.gsfc.nasa.gov/stargaze/Kep3laws.htm

Kepler's Third Law Calculator: http://www.1728.org/kepler3.htm

Venus Again

Halley on Venus Parallax: https://eclipse.gsfc.nasa.gov/transit/HalleyParallax.html

Captain James Cook and the 1769 Transit of Venus: https://science.nasa.gov/science-news/science-at-nasa/2004/28may_cook and https://science.nasa.gov/science-news/science-at-nasa/2012/02jun_jamescook

University of Pittsburgh:
https://sites.pitt.edu/~jdnorton/papers/material_large/12_Distances_solar_system.pdf

ESA blog: https://blogs.esa.int/venustransit/2012/05/30/measuring-the-size-of-the-solar-system-parallax

Any You Think YOU Had a Bad Day? Tell it to LeGentil:
https://www.messier.seds.org/xtra/Bios/legentil.html! [But he played a mean (bass) guitar: https://tabs.ultimate-guitar.com/tab/rob-scallon/le-gentil-the-transit-of-venus-guitar-pro-1425271 (tabs) – YouTube:
https://www.youtube.com/watch?v=njoqfwd7vn4&ab_channel=RobScallon

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